



A mathematical modeling of fluid flow through a Wavy Walled square pipe in environmental studies: Non-orthogonal coordinates

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Abstract: In this paper we consider the steady flow of a viscous fluid through a rectilinear pipe bounded by four sinusoidally varying plates in same phase with opposite plates separated by a mean distance $2h$. For the non-varying rectilinear pipe with rectangular cross section, the classical velocity profile for the fully developed flow is well known. In the present research an attempt is made to analyze the flow in a generalized non-orthogonal coordinate system that renders the wavy pipe as plane walled pipe. Continuity equation and Navier-stokes equations are presented in the generalized coordinate system and simplified through use of small perturbation under small Reynolds number approximation. Flow characteristics such as centerline velocity have been evaluated and discussed. The results of the paper have possible applications in flow of air pollutants in stakes and waste water through pipes and also in life sciences.

Key words: Wavy pipe, Navier-stoke equations, Small Reynolds number, Perturbation

Introduction

Fluid transport through pipes is not only being used in human endeavors but also occurs extensively in nature. We may cite the transport of water, petrol and other chemicals for sustaining life and its betterment. As a consequence of worldwide pollution control transport of waste products like smoke, exhaust gases and sludge through conduits is becoming a factor of major importance as there is complete absence of release of pollutants in the atmosphere. The flow of blood and other body fluids in human beings and animals, xylem and phloem flow and flow through brush like plants are examples of fluid transport in ecosystem. Percolation and seepage of water and oil through crevices, faults and limestone aquifers exemplifies pipe transport in geophysical problems. The cross sections of the pipes are of various geometries – circular, elliptical, square *etc.*, but because of factors such as corrosion, biofilm deposit the generating lengths of the pipes are seldom smooth straight lines. For mathematical modeling it becomes convenient to approximate these by sinusoidal curves. The rectilinear pipe bounded by four wavy plates has possible applications in oil recovery, biological transport processes, polymeric composite manufacturing and for enhancement of heat transfer in heat exchangers.

Flow in sinusoidal wavy wall has been analyzed amongst others by Benjamin (1959), Kim (2001), Lyne (1971), Montalbano and Mc Cready (1998), Patel *et al.* (1991), Ralph (1987) and Wang (2004). Sankar and Sinha (1976) studied the Rayleigh problem for a wavy wall by applying conformal mapping technique to transform the wavy boundary to a regular one. Aaron *et al.* (2006), neglecting inertia terms and using perturbation method that transforms a problem with irregular boundaries into a series of problems with plane boundaries, have investigated the flow profile and power requirements in an oscillatory flow field in a wavy walled channel.

In this paper we model pipe flow in a square sectioned pipe with sinusoidal walls with opposite wall in same phase. This paper is different from others as here generalized non-orthogonal coordinate system is used rendering directly the wavy walls to plane ones. The method may be extended from sinusoidally varying walls to walls with arbitrary fluctuation.

In generalized coordinate system with coordinates x^i ($i = 1, 2$), the equation of continuity for incompressible steady flow is given by

$$\frac{\partial v^i}{\partial x^i} + v^k \left\{ \begin{matrix} i \\ k \ i \end{matrix} \right\} = 0 \quad \dots\dots\dots (1.1)$$

and Navier-Stokes equations with no external force are given by Sokolnikoff (1964):

$$v^j \left(\frac{\partial v^i}{\partial x^j} + \left\{ \begin{matrix} i \\ l j \end{matrix} \right\} v^l \right) = g^{jk} \left(\frac{\mu}{\rho} \right) \left[\frac{\partial^2 v^i}{\partial x^j \partial x^k} + \left\{ \begin{matrix} i \\ l k \end{matrix} \right\} \frac{\partial v^l}{\partial x^j} + \left\{ \begin{matrix} i \\ l j \end{matrix} \right\} \frac{\partial v^l}{\partial x^k} - \left\{ \begin{matrix} l \\ j k \end{matrix} \right\} \frac{\partial v^i}{\partial x^l} \right] \dots\dots(1.2)$$

$$+ g^{jk} \left(\frac{\mu}{\rho} \right) \left[\frac{\partial}{\partial x^k} \left\{ \begin{matrix} i \\ l j \end{matrix} \right\} + \left\{ \begin{matrix} i \\ m k \end{matrix} \right\} \left\{ \begin{matrix} m \\ l j \end{matrix} \right\} - \left\{ \begin{matrix} i \\ l m \end{matrix} \right\} \left\{ \begin{matrix} m \\ j k \end{matrix} \right\} \right] v^l - \frac{g^{ik}}{\rho} \frac{\partial p}{\partial x^k}$$

where v^i ($i=1,2$) are contravariant components of velocity. p is the pressure, μ is coefficient of viscosity, ρ is the density of fluid at any point and also curly bracket $\{ \}$ represents Christoffel symbol of second kind .

Equations governing incompressible wavy pipe flow in generalized coordinates: Now, consider the flow through a pipe whose wall are four wavy surfaces.

$$\left. \begin{aligned} Y &= \left[\pm 1 + \delta \text{Sin}(\lambda h x^1) \right] \\ Z &= \left[\pm 1 + \delta \text{Sin}(\lambda h x^1) \right] \end{aligned} \right\} \dots\dots(2.1)$$

where $2h$ is the mean distance between the two opposite surfaces, λ is 2π times the wave number and (X,Y) are Cartesian coordinates with X - axis along the straight central line of the pipe.

We introduce the transformation

$$\left. \begin{aligned} X &= h x^1 \\ Y &= h \left[x^2 + \delta \text{Sin}(\lambda h x^1) \right] \\ Z &= h \left[x^3 + \delta \text{Sin}(\lambda h x^1) \right] \end{aligned} \right\} \dots\dots(2.2)$$

so that the pipe boundaries assume the simple form $x^2 = \pm 1$ and $x^3 = \pm 1$.

Observe that x^i ($i=1,2$) are non-orthogonal curvilinear coordinates. For convenience, we denote

$$\text{Sin}(\lambda h x^1) = S, \text{Cos}(\lambda h x^1) = C \text{ and } \lambda h = \Lambda .$$

The metric tensor g^{ij} and g_{ij} are then obtained as

$$g_{ij} = \begin{pmatrix} h^2 & h^2 \delta \Lambda C & h^2 \delta \Lambda C \\ h^2 \delta \Lambda C & h^2 & 0 \\ h^2 \delta \Lambda C & 0 & h^2 \end{pmatrix}$$

$$g^{ij} = \begin{pmatrix} h^{-2} & -h^{-2} \delta \Lambda C & -h^{-2} \delta \Lambda C \\ -h^{-2} \delta \Lambda C & h^{-2} & 0 \\ -h^{-2} \delta \Lambda C & 0 & h^{-2} \end{pmatrix} \dots\dots(2.3)$$

$$g = |g_{ij}| = h^6 .$$

Using tensor methods, assuming δ to be a small perturbation and neglecting $O(\delta^2)$ terms, equation of continuity and Navier–Stokes equations for steady flow become

$$\frac{\partial v^1}{\partial x^1} + \frac{\partial v^2}{\partial x^2} + \frac{\partial v^3}{\partial x^3} = 0 \quad \dots\dots(2.4)$$

$$v^1 \frac{\partial v^1}{\partial x^1} + v^2 \frac{\partial v^1}{\partial x^2} + v^3 \frac{\partial v^1}{\partial x^3} = \frac{\mu}{\rho h^2} \left\{ \frac{\partial^2 v^1}{(\partial x^1)^2} + \frac{\partial^2 v^1}{(\partial x^2)^2} + \frac{\partial^2 v^1}{(\partial x^3)^2} - 2\delta\Lambda C \left(\frac{\partial^2 v^1}{\partial x^2 \partial x^1} + \frac{\partial^2 v^1}{\partial x^3 \partial x^1} \right) + \delta\Lambda^2 S \left(\frac{\partial v^1}{\partial x^2} + \frac{\partial v^1}{\partial x^3} \right) \right\} + \frac{1}{\rho h^2} \left(-\frac{\partial p}{\partial x^1} + \delta\Lambda C \frac{\partial p}{\partial x^2} + \delta\Lambda C \frac{\partial p}{\partial x^3} \right) \quad \dots\dots(2.5)$$

$$\left\{ v^1 \frac{\partial v^2}{\partial x^1} - \delta\Lambda^2 S (v^1)^2 \right\} - \left\{ \frac{(\delta\Lambda C)}{\rho h^2} \frac{\partial p}{\partial x^1} - \frac{1}{\rho h^2} \frac{\partial p}{\partial x^2} \right\} = h^{-2} \left(\frac{\mu}{\rho} \right) \left\{ \frac{\partial^2 v^2}{(\partial x^1)^2} + \frac{\partial^2 v^2}{(\partial x^2)^2} + \frac{\partial^2 v^2}{(\partial x^3)^2} - 2\delta\Lambda^2 S \frac{\partial v^1}{\partial x^1} - \delta\Lambda^3 C v^1 \right\} \quad \dots\dots(2.6)$$

$$\left\{ v^1 \frac{\partial v^3}{\partial x^1} - \delta\Lambda^2 S (v^1)^2 \right\} - \left\{ \frac{(\delta\Lambda C)}{\rho h^2} \frac{\partial p}{\partial x^1} - \frac{1}{\rho h^2} \frac{\partial p}{\partial x^3} \right\} = h^{-2} \left(\frac{\mu}{\rho} \right) \left\{ \frac{\partial^2 v^3}{(\partial x^1)^2} + \frac{\partial^2 v^3}{(\partial x^2)^2} + \frac{\partial^2 v^3}{(\partial x^3)^2} - 2\delta\Lambda^2 S \frac{\partial v^1}{\partial x^1} - \delta\Lambda^3 C v^1 \right\} \quad \dots\dots(2.7)$$

These equations have to be solved under the no slip condition and vanishing of the normal velocity at $x^2 = \pm 1$ and $x^3 = \pm 1$.

Perturbation solution: Assuming to be small and introducing dimensionless new variables $u^0, u^1, w_2^1, w_3^1, p^0$ and p^1 as follows

$$\left. \begin{aligned} v^1 &= \frac{U}{h} (u^0 + \delta u^1) \\ v^2 &= \frac{U}{h} \delta w_2^1 \\ v^3 &= \frac{U}{h} \delta w_3^1 \\ p &= (p^0 + \delta p^1) \rho U^2 \end{aligned} \right\} \quad \dots\dots(3.1)$$

Observe that the contravariant velocity components, v^1, v^2 and v^3 have same dimensions as $\frac{U}{h}$ and pressure p has same dimension as ρU^2 Here U represents the centerline velocity corresponding to straight channel which is linearly related to the corresponding pressure gradient. Substituting (3.1) in equation (2.4), (2.5), (2.6) and (2.7) we get two sets of four equations as follows:

i. Zero order equation:

$$\left. \begin{aligned} \frac{\partial u^0}{\partial x^1} &= 0 \\ \frac{\partial p^0}{\partial x^1} &= \frac{1}{R} \left\{ \frac{\partial^2 u^0}{(\partial x^2)^2} + \frac{\partial^2 u^0}{(\partial x^3)^2} \right\} \\ \frac{\partial p^0}{\partial x^2} &= 0 \\ \frac{\partial p^0}{\partial x^3} &= 0 \end{aligned} \right\} \dots\dots(3.2)$$

where $R = \frac{\rho h U}{\mu}$ is Reynolds number.

The zero order solution with the no slip boundary conditions $u^0(\pm 1, x^3)$ and $u^0(x^2, \pm 1)$ provides

$$u^0 = \left(-\frac{R}{2} \frac{\partial p^0}{\partial x^1} \right) \left[\left\{ 1 - (x^3)^2 \right\} + 4 \sum_{n=1}^{\infty} \left\{ \frac{(-1)^n \text{Cosh}(\alpha_n x^2) \text{Cos}(\alpha_n x^3)}{(\alpha_n)^3 \text{Cosh}(\alpha_n)} \right\} \right]$$

It may be noted that we get the same expression u^0 on interchanging x^2 and x^3 , and hence on averaging we get in symmetrical form

$$u^0 = \left(-\frac{R}{4} \frac{\partial p^0}{\partial x^1} \right) \left[\left\{ 2 - (x^2)^2 - (x^3)^2 \right\} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{\text{Cosh}(\alpha_n)} \left(\frac{\text{Cosh}(\alpha_n x^2) \text{Cos}(\alpha_n x^3) + \text{Cosh}(\alpha_n x^3) \text{Cos}(\alpha_n x^2)}{(\alpha_n)^3} \right) \right] \dots\dots(3.3)$$

$$\left(\frac{\partial p^0}{\partial x^2} \right) = 0 \dots\dots(3.4)$$

$$\left(\frac{\partial p^0}{\partial x^3} \right) = 0 \dots\dots(3.5)$$

$$\left(\frac{\partial p^0}{\partial x^1} \right) = -\frac{2}{R} \left[1 + 4 \sum_{n=1}^{\infty} \left\{ \frac{(-1)^n}{(\alpha_n)^3 \text{Cosh}(\alpha_n)} \right\} \right]^{-1} \dots\dots(3.6)$$

where $\alpha_n = \frac{(2n-1)\pi}{2}$.

ii. **First order perturbation equations:** Using solution [(3.3) - (3.6)], the first order equations are expressible as:

$$\frac{\partial u^1}{\partial x^1} + \frac{\partial w_2^1}{\partial x^2} + \frac{\partial w_3^1}{\partial x^3} = 0 \quad \dots\dots(3.7)$$

$$\left[u^0 \frac{\partial u^1}{\partial x^1} + w_2^1 \frac{\partial u^0}{\partial x^2} + w_3^1 \frac{\partial u^0}{\partial x^3} \right] = \frac{\mu}{\rho h U} \left\{ \frac{\partial^2 u^1}{(\partial x^1)^2} + \frac{\partial^2 u^1}{(\partial x^2)^2} + \frac{\partial^2 u^1}{(\partial x^3)^2} + \Lambda^2 S \left(\frac{\partial u^0}{\partial x^2} + \frac{\partial u^0}{\partial x^3} \right) \right\} + \left(-\frac{\partial p^1}{\partial x^1} \right) \quad \dots\dots(3.8)$$

$$\left\{ u^0 \frac{\partial w_2^1}{\partial x^1} - \Lambda^2 S (u^0)^2 \right\} + \left\{ \frac{2\Theta \Lambda C}{R} + \frac{\partial p^1}{\partial x^2} \right\} = \frac{1}{R} \left\{ \frac{\partial^2 w_2^1}{(\partial x^1)^2} + \frac{\partial^2 w_2^1}{(\partial x^2)^2} + \frac{\partial^2 w_2^1}{(\partial x^3)^2} - \Lambda^3 C u^0 \right\} \quad \dots\dots(3.9)$$

$$\left\{ u^0 \frac{\partial w_3^1}{\partial x^1} - \Lambda^2 S (u^0)^2 \right\} + \left\{ \frac{2\Theta \Lambda C}{R} + \frac{\partial p^1}{\partial x^3} \right\} = \frac{1}{R} \left\{ \frac{\partial^2 w_3^1}{(\partial x^1)^2} + \frac{\partial^2 w_3^1}{(\partial x^3)^2} + \frac{\partial^2 w_3^1}{(\partial x^2)^2} - \Lambda^3 C u^0 \right\} \quad \dots\dots(3.10)$$

where $\left(-\frac{R}{2} \frac{\partial p^0}{\partial x^1} \right) = \Theta$.

These equations have been solved under following boundary conditions:

$$v^1 = 0 \text{ and } v^2 = 0 \text{ at } x^2 = \pm 1.$$

For small values of Reynolds numbers R, the solution is

$$v^1 = \left(\frac{U}{h} \right) \left[u^0 + \delta \left[\frac{2\Theta_0}{\Lambda + h_c h_s} \left(e_3 \{ h_c (x^2) \Lambda \text{Cosh}(\Lambda x^2) + (h_c - h_s \Lambda) \text{Sinh}(\Lambda x^2) \} + e_2 \{ h_c (x^3) \Lambda \text{Cosh}(\Lambda x^3) + (h_c - h_s \Lambda) \text{Sinh}(\Lambda x^3) \} \right) - \Theta \{ (x^2) + (x^3) + 2\Sigma_1 \} \right] \text{Sin}(\Lambda x^1) \right]$$

where

$$\Sigma_1 = \sum_{n=1}^{\infty} \left(\frac{\text{Sinh}(\alpha_n x^2) \text{Cos}(\alpha_n x^3) - \text{Cosh}(\alpha_n x^3) \text{Sin}(\alpha_n x^2) + \text{Sinh}(\alpha_n x^3) \text{Cos}(\alpha_n x^2) - \text{Cosh}(\alpha_n x^2) \text{Sin}(\alpha_n x^3)}{(-1)^n (\alpha_n)^2 \text{Cosh}(\alpha_n)} \right) \quad \dots\dots(3.11)$$

$$v^2 = \delta \Lambda \left(\frac{U}{h} \right) \left[2\Theta_0 \left\{ \frac{e_3}{\Lambda + h_c h_s} \right\} \left\{ h_s \text{Cosh}(\Lambda x^2) - h_c (x^2) \text{Sinh}(\Lambda x^2) \right\} - u^0 \right] \text{Cos}(\Lambda x^1) \quad \dots\dots(3.12)$$

$$v^3 = \delta \Lambda \left(\frac{U}{h} \right) \left[2\Theta_0 \left\{ \frac{e_2}{\Lambda + h_c h_s} \right\} \left\{ h_s \text{Cosh}(\Lambda x^3) - h_c (x^3) \text{Sinh}(\Lambda x^3) \right\} - u^0 \right] \text{Cos}(\Lambda x^1).$$

where $\Theta_0 = \frac{\Theta}{2} \left\{ 1 - 2 \sum_{n=1}^{n=\infty} \frac{(-1)^n}{\text{Cosh}(\alpha_n)} \left(\frac{\text{Sinh}(\alpha_n) - \text{Sin}(\alpha_n)}{(\alpha_n)^2} \right) \right\}$, $\alpha_n = \frac{(2n-1)\pi}{2}$, $h_c = \text{Cosh}(\Lambda)$

$$h_s = \text{Sinh}(\Lambda), e_2 = \sum_{n=1}^{n=\infty} \frac{4(-1)^{n-1}}{(2n-1)\pi} \text{Cos}(\alpha_n x^2) \text{ and } e_3 = \sum_{n=1}^{n=\infty} \frac{4(-1)^{n-1}}{(2n-1)\pi} \text{Cos}(\alpha_n x^3).$$

Conclusion

For obtaining the physical results the solution has been transformed to the non-orthogonal Cartesian components, which up to first order perturbation in δ are as follows

$$V_c(1) = U \left[u^0 + \delta \left[\frac{2\Theta_0}{\Lambda + h_c h_s} \left(e_3 \left\{ h_c \left(\Lambda \frac{Y}{h} \right) \text{Cosh} \left(\Lambda \frac{Y}{h} \right) + (h_c - h_s \Lambda) \text{Sinh} \left(\Lambda \frac{Y}{h} \right) \right\} + e_2 \left\{ h_c \left(\Lambda \frac{Z}{h} \right) \text{Cosh} \left(\Lambda \frac{Z}{h} \right) + (h_c - h_s \Lambda) \text{Sinh} \left(\Lambda \frac{Z}{h} \right) \right\} - \Theta \left(\frac{Y}{h} + \frac{Z}{h} + 2\Sigma_1 \right) \right] \text{Sin} \left(\Lambda \frac{X}{h} \right) \right] \quad \dots\dots(3.13)$$

$$V_c(2) = 2\delta U \Lambda \Theta_0 \left\{ \frac{e_3}{\Lambda + h_c h_s} \right\} \left\{ h_s \text{Cosh} \left(\Lambda \frac{Y}{h} \right) - h_c \left(\frac{Y}{h} \right) \text{Sinh} \left(\Lambda \frac{Y}{h} \right) \right\} \text{Cos} \left(\Lambda \frac{X}{h} \right) \quad \dots\dots(3.14)$$

$$V_c(3) = 2\delta U \Lambda \Theta_0 \left\{ \frac{e_2}{\Lambda + h_c h_s} \right\} \left\{ h_s \text{Cosh} \left(\Lambda \frac{Z}{h} \right) - h_c \left(\frac{Z}{h} \right) \text{Sinh} \left(\Lambda \frac{Z}{h} \right) \right\} \text{Cos} \left(\Lambda \frac{X}{h} \right) \quad \dots\dots(3.15)$$

where

$$\Sigma_1 = \sum_{n=1}^{n=\infty} \left(\frac{\text{Sinh} \left(\alpha_n \left(\frac{Y}{h} \right) \right) \text{Cos} \left(\alpha_n \left(\frac{Z}{h} \right) \right) - \text{Cosh} \left(\alpha_n \left(\frac{Y}{h} \right) \right) \text{Sin} \left(\alpha_n \left(\frac{Z}{h} \right) \right)}{(-1)^n (\alpha_n)^2 \text{Cosh}(\alpha_n)} \right) + \sum_{n=1}^{n=\infty} \left(\frac{\text{Sinh} \left(\alpha_n \left(\frac{Z}{h} \right) \right) \text{Cos} \left(\alpha_n \left(\frac{Y}{h} \right) \right) - \text{Cosh} \left(\alpha_n \left(\frac{Z}{h} \right) \right) \text{Sin} \left(\alpha_n \left(\frac{Y}{h} \right) \right)}{(-1)^n (\alpha_n)^2 \text{Cosh}(\alpha_n)} \right) \quad \dots\dots(3.16)$$

$$u^0 = -\frac{R}{4} \frac{\partial p^0}{\partial x^1} \left[2 - \left(\frac{Y}{h}\right)^2 - \left(\frac{Z}{h}\right)^2 + 4\Sigma_2 + 4\delta(1 + \Sigma_3 - \Sigma_4)S \right] \quad \text{.....(3.17)}$$

$$\Sigma_2 = \sum_{n=1}^{\infty} \left(\frac{\left[\text{Cosh}\left(\frac{\alpha_n Y}{h}\right) \text{Cos}\left(\frac{\alpha_n Z}{h}\right) + \text{Cosh}\left(\frac{\alpha_n Z}{h}\right) \text{Cos}\left(\frac{\alpha_n Y}{h}\right) \right]}{(-1)^n (\alpha_n)^3 \text{Cosh}(\alpha_n)} \right) \quad \text{.....(3.18)}$$

$$\Sigma_3 = \sum_{n=1}^{\infty} \left(\frac{\left[\text{Cosh}\left(\frac{\alpha_n Y}{h}\right) \text{Sin}\left(\frac{\alpha_n Z}{h}\right) + \text{Cosh}\left(\frac{\alpha_n Z}{h}\right) \text{Sin}\left(\frac{\alpha_n Y}{h}\right) \right]}{(-1)^n (\alpha_n)^2 \text{Cosh}(\alpha_n)} \right) \quad \text{.....(3.19)}$$

$$\Sigma_4 = \sum_{n=1}^{\infty} \left(\frac{\left[\text{Sinh}\left(\frac{\alpha_n Z}{h}\right) \text{Cos}\left(\frac{\alpha_n Y}{h}\right) + \text{Sinh}\left(\frac{\alpha_n Y}{h}\right) \text{Cos}\left(\frac{\alpha_n Z}{h}\right) \right]}{(-1)^n (\alpha_n)^2 \text{Cosh}(\alpha_n)} \right) \quad \text{.....(3.20)}$$

Center line velocity: Letting $Y = Z = \delta h S$ in [(3.13) - (3.16)], neglecting $O(\delta^2)$ terms, we get axial and transverse components of central line velocity as

$$\begin{aligned} V_x(c) &= U \\ V_y(c) &= \left\{ \frac{2\delta U \Lambda \Theta_0 h_s}{\Lambda + h_c h_s} \right\} \text{Cos}\left(\frac{\Lambda X}{h}\right) \\ \text{and } V_z(c) &= V_y(c) \end{aligned} \quad \text{.....(3.21)}$$

respectively. It is observed that axial component has the same constant value as in the corresponding straight pipe case. In the figures below we exhibit the variations in these components. There is no loss of generality in taking $h=1$ so that $x^1=X$ and $\lambda = \Lambda$.

In Fig., below, we have taken $\Lambda = 1$ and $\delta = 0.1$. This figure shows the relation (3.21) of non-dimensional Cartesian velocity components $\frac{V_x(c)}{U}$ and $\frac{V_y(c)}{U}$ with X coordinates for $x^2 = 0$. Cartesian Component of velocity $\frac{V_x(c)}{U}$ for $x^2=0$ i.e. $\left(\frac{Y}{h} = \delta S\right)$ is same as velocity component along corresponding straight pipe while $\frac{V_y(c)}{U}$ varies sinusoidally with X having phase difference $\frac{\pi}{2}$ with wall waviness.

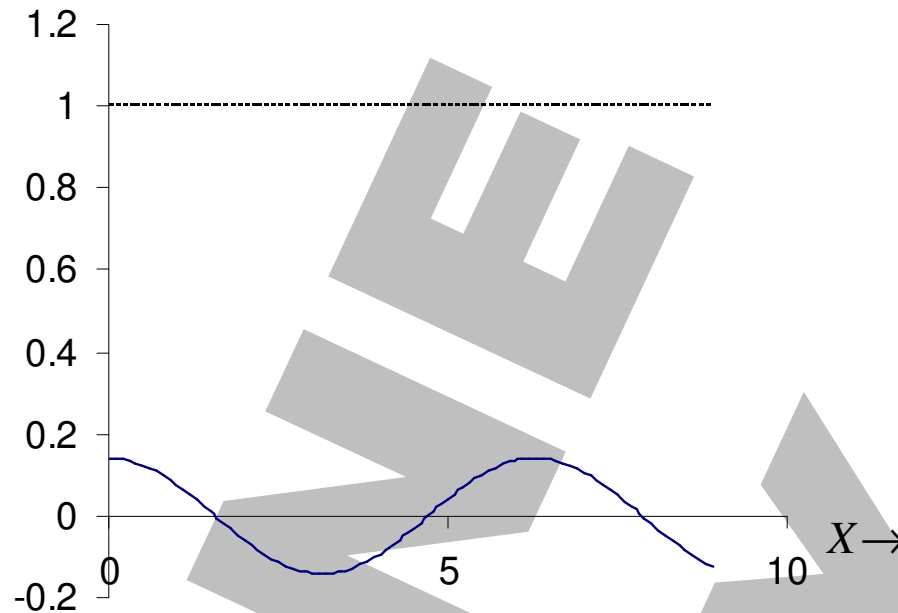


Fig. 1: Variation in non-dimensional Cartesian Velocity Components

$$\frac{V_x(c)}{U} \dots\dots\dots \text{and} \frac{V_y(c)}{U} \sim \text{with } X \text{ for } x^2 = 0$$

It is also noticed that $\frac{V_y(c)}{U} = \frac{V_x(c)}{U}$ because $\frac{V_y(c)}{V_x(c)} = \delta \left[\frac{2\Theta_0 h_s}{\left(1 + \frac{h_c h_s}{\Lambda}\right)} \text{Cos} \left(\frac{\Lambda X}{h} \right) \right] < 0.142$.

We are in the process to obtain the other mathematical aspects e.g. streamline and drag force and ascertaining the values of the parameters involved in the analysis. The out come of these shall yield interesting results of practical utility and will be discussed in a forthcoming paper.

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